# Finite Math - Fall 2016

Lecture Notes - 11/9/2016

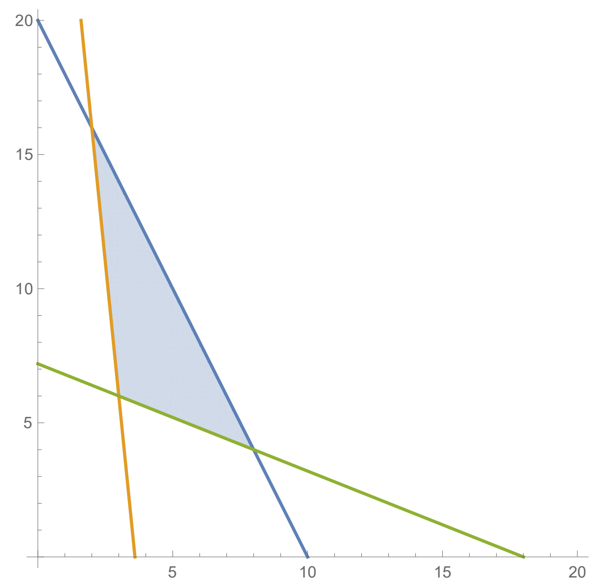
# Section 5.3 - Linear Programming in Two Dimensions: A Geometric Approach

## Geometric Method for Solving Linear Programming Problems.

**Example 1.** *Maximize and minimize* $z = 3x + y$ *subject to the inequalities*

$$\begin{aligned} 2x + y &\leq 20 \\ 10x + y &\geq 36 \\ 2x + 5y &\geq 36 \\ x, y &\geq 0 \end{aligned}$$

**Solution.** *We begin by graphing the feasible region*




*Since this is a bounded region, by part (A) of Theorem 2, this problem has both a maximum and a minimum value. Now we find the corner points, of which there are three.*

| *colors* | *system* | *corner point* |
|---|---|---|
| *blue and orange* | $\begin{cases} 2x + y = 20 \\ 10x + y = 36 \end{cases}$ | $(2, 16)$ |
| *blue and green* | $\begin{cases} 2x + y = 20 \\ 2x + 5y = 36 \end{cases}$ | $(8, 4)$ |
| *orange and green* | $\begin{cases} 10x + y = 36 \\ 2x + 5y = 36 \end{cases}$ | $(3, 6)$ |

*Now, we check the value of* $z$ *at these corner points:*

| *corner point* | $z = 3x + y$ |
|---|---|
| $(2, 16)$ | $z = 3(2) + (16) = 6 + 16 = 22$ |
| $(8, 4)$ | $z = 3(8) + (4) = 24 + 4 = 28$ |
| $(3, 6)$ | $z = 3(3) + (6) = 9 + 6 = 15$ |

*Then, we see that the minimum value is* $15$ *and occurs at* $(3, 6)$ *and the maximum value is* $28$ *and occurs at* $(8, 4)$.

**Example 2.** *Maximize and minimize* $z = 2x + 3y$ *subject to*

$$\begin{aligned} x - 2y &\geq 0 \\ 2x - y &\leq 6 \\ x + y &\geq 3 \\ x, y &\geq 0 \end{aligned}$$

**Solution.** *Minimum of* $z = 6$ *at* $(3, 0)$ *and maximum of* $z = 14$ *at* $(4, 2)$.

**Applications.**

**Example 3.** *An electronics firm manufactures two types of personal computers–a standard model and a portable model. The production of a standard computer requires a capital expenditure of* \$400 *and* 40 *hours of labor. The production of a portable computer requires a capital expenditure of* \$250 *and* 30 *hours of labor. The firm has* \$20,000 *capital and* 2,160 *labor-hours available for production of standard and portable computers.*

(a) *What is the maximum number of computers the company is capable of producing?*

(b) *If each standard computer contributes a profit of* \$320 *and each portable model contributes a profit of* \$220*, how much profit will the company make by producing the maximum number of computers?*

(c) *Does producing as many computers as possible produce the highest profit? If not, what is the highest profit and how many of each computer should be made in that case?*

**Solution.** *Let* $x = \#$ *of standard model and* $y = \#$ *of portable model. Let's find the feasible region for this problem before continuing. From capital, we get the inequality*

$$400x + 250y \leq 20000$$

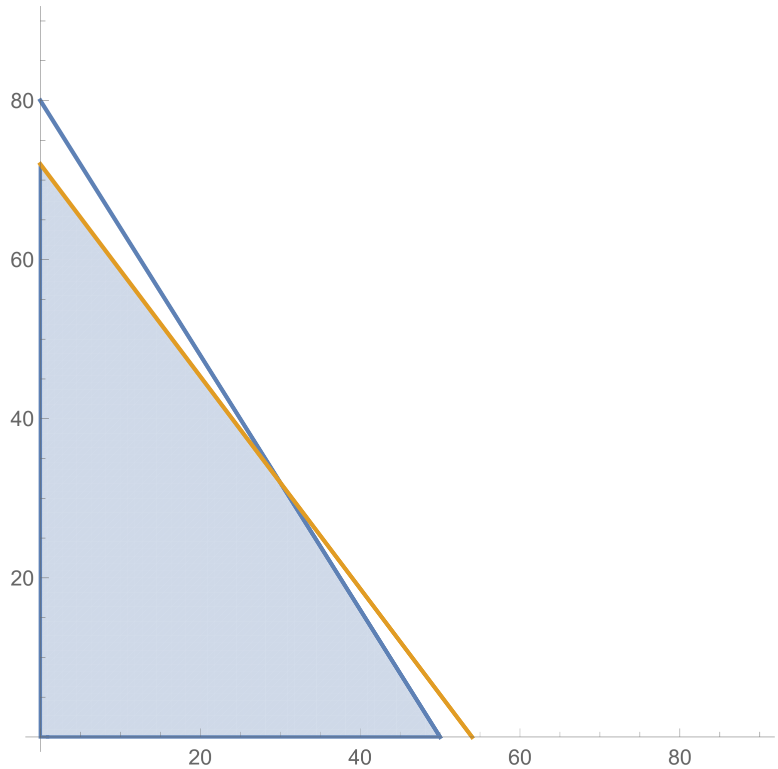
*and from labor-hours we get*

$$40x + 30y \leq 2160$$

*and of course we add in*

$$x, y \geq 0$$

*since a negative number of computers cannot be produced. The graph of the feasible region is*




*Observe that the feasible region is bounded. The corner points of the feasible region are*

$$(0, 0), (0, 72), (50, 0), (30, 32).$$

(a) *The amount of computers produced is simply*

$$C = x + y$$

*so to figure out the maximum number of computers the company is capable of producing we just have to maximize $C$. Test $C$ at the corner points*

| *Corner Point* | *$C$ value* |
|---|---|
| $(0, 0)$ | 0 |
| $(0, 72)$ | 72 |
| $(50, 0)$ | 50 |
| $(30, 32)$ | 62 |

*So to produce the largest amount of computers, they should produce* 72 *portable computers and no standard models.*

(b) *The profit function is*

$$P = 320x + 220y.$$

*By producing the largest amount of computers, the company would make*

$$320(0) + 220(72) = 15840 \ \textit{dollars}$$

(c) *To check if this gives the highest profit, we should check the profit function at all of the corner points.*

| *Corner Point* | *Profit* |
|---|---|
| $(0, 0)$ | $\$0$ |
| $(0, 72)$ | $\$15,840$ |
| $(50, 0)$ | $\$16,000$ |
| $(30, 32)$ | $\$16,640$ |

*So we see that producing* 30 *standard models and* 32 *portable models will produce the highest profit for the company.*

**Example 4.** *A fruit grower can use two types of fertilizer in his orange grove, brand A and brand B. The amounts (in pounds) of nitrogen, phosphoric acid, and chloride in a bag of each brand are given in the table. Tests indicate that the grove needs at least* $1,000$ *pounds of phosphoric acid and at most* 400 *pounds of chloride.*

| | *Brand A* | *Brand B* |
|---|---|---|
| *Nitrogen* | 8 | 3 |
| *Phosphoric Acid* | 4 | 4 |
| *Chloride* | 2 | 1 |

(a) *If the grower wants to maximize the amount of nitrogen added to the grove, how many bags of each mix should be used? How much nitrogen will be added?*

(b) *If the grower wants to minimize the amount of nitrogen added to the grove, how many bags of each mix should be used? How much nitrogen will be added?*

**Solution.**

(a) 150 *bags brand A,* 100 *bags brand B,* $1,500$ *lbs of nitrogen*

(b) 0 *bags brand A,* 250 *bags brand B,* 750 *lbs of nitrogen*